# FLUID TO FLUID MODELING OF CRITICAL HEAT FLUX: A COMPENSATED DISTORTION MODEL

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Abstract—Modeling is of practical importance in reducing the high cost of critical heat flux testing for boiling water power reactors. A method widely used is to replace water by a modeling fluid having a lower latent heat of vaporization. In this paper a generalized technique for such fluid modeling of critical heat flux is developed from classical dimensional analysis and theory of models. The problem of multiple distortion is solved by introducing a modeling parameter which is a function of Weber number and liquid and vapor superficial Reynolds numbers. The technique of compensated distortion is employed in the development of the modeling parameter.

The resulting model has been tested at different liquid/vapor density ratios for various Freon compounds, water, potassium, and carbon dioxide, over a large range of mass flux and inlet sub-cooling, and in both the subcooled and quality regions. Excellent agreement is shown between experimental data and the modeling technique for circular, annular, and rod bundle geometries.

	NOMENCLATURE	М,	mass;
а, b–е,	constants;	$n_1, n_2,$	empirical constants;
$C_1, C_2, C_3$	$C_3$ , constants;	Ρ,	absolute pressure force per unit
Cp,	heat capacity (specific);		area;
CHF,	critical heat flux;	Т,	time;
<b>D</b> ,	hydraulic diameter (= $4 \times $ flow	$W^+,$	reduced power number defined in
	area/wetted perimeter);		equation (20);
Dhe,	heated equivalent diameter (= $4 \times$	$X_{in}$ ,	vapor fraction by weight at inlet;
	flow area/heated perimeter);	$X_0,$	vapor fraction by weight at outlet;
Ε,	dimensionless parameter defined	y <sup>+</sup> ,	dimensionless parameter defined
	in equation (23b);		in equation (22b).
<b>E</b> <sup>+</sup> ,	dimensionless parameter defined		
	in equation (23a);	Greek sym	bols
$F_D$ ,	size scaling factor;	θ,	temperature;
$F_{G}$ ,	mass flux scaling factor;	ho,	density;
<i>g</i> ,	gravitational acceleration;	μ,	dynamic viscosity;
<i>G</i> ,	mass-transfer rate per unit area;	σ,	surface tension;
Η,	enthalpy;	$\phi,$	heat transfer rate per unit area
$H_L$ ,	enthalpy of saturated liquid;		(critical heat flux);
$H_{in},$	enthalpy of flowing medium at	λ,	latent heat of vaporization;
	inlet;	$\psi_{\mathrm{CHF}},$	modeling parameter defined in
$\Delta H$ ,	inlet subcooling $(=H_L - H_{in});$		equation (13);
Κ,	thermal conductivity;		$\left \partial(a_{1}(a_{2}))\right $
K <sub>s</sub> ,	scaling constant $(= 1/F_G)$ ;	γ,	$\left \frac{\nabla(\mathcal{P}L/\mathcal{P}V)}{2\mathcal{P}}\right $ saturation;
L,	length and heated length;		

$$\beta, \qquad \left| \frac{\partial \theta}{\partial P} \right| \text{ saturation}; \\ \pi_1 \dots \pi_{13} \quad \text{dimensionless Pi groups defined} \\ \text{ in equation (2).}$$

**Subscripts** 

L,	liquid phase:	
<i>V</i> ,	vapor phase.	

Note

 $\operatorname{Error}(\%) = 100 \left[ \frac{\phi_{\text{measured}} - \phi_{\text{predicted}}}{\phi_{\text{measured}}} \right]$  $= 100 \left[ \frac{W_{\text{measured}}^{+} - W_{\text{predicted}}^{+}}{W_{\text{measured}}^{+}} \right]$  $\left\{ (W_{\text{measured}}^{+}) \text{ calculated from equation (20)} \right\}$  $\left\{ (W_{\text{predicted}}^{+}) \text{ calculated from equation (22)} \right\}$  $\operatorname{rms error} = \sqrt{\left( \frac{\Sigma \text{ Error}^{2}}{\text{No. of observations}} \right)}.$ 

#### **INTRODUCTION**

FLUID modeling is a technique by which thermal and hydraulic behavior of a given system can be studied using an expedient fluid rather than the working fluid.

Current interest in fluid modeling arises primarily because of problems in thermal and hydraulic design of nuclear reactors. Dryout is one such problem; it is a phenomenon that occurs in boiling-water-cooled fuel channels when there is a depletion of liquid film which normally flows over the fuel. Consequently the efficient process of boiling heat transfer breaks down and the fuel sheath experiences an abrupt rise in temperature. As this may prove a limitation to fuel performance it is important for nuclear power plant designers to be able to predict the critical heat flux (CHF) and other system parameters at which dryout occurs.

Because of the complex nature of the problem, attempts to formulate a theoretical model for predicting CHF for multi-rod bundles have not been very successful. Costly *ad hoc* experimentation with the working fluid and with full-scale geometries is used to acquire CHF data for the development of empirical design correlations.

To reduce the high electrical power cost, parameter scaling has been attempted. One method is to scale down the size of test sections, while using the same working coolant, e.g. water. This form of scaling has been successful for simple geometries, but there is evidence that it does not work well for multi-rod bundles.

An alternative method—which has been widely used—is to replace the working fluid (e.g. water) by a modeling fluid having a lower latent heat of vaporization. This reduces the test section power considerably. The fluids usually chosen for such modeling are members of the fluorocarbon family (Freons, Arctons) because of their very low latent heat, relatively low hazard and well established fluid properties. Furthermore, the required identical liquid/vapor density ratio for both water and Freon results in much lower operating pressures and temperatures for Freons. This allows improved visualization capability, and reduces loop construction costs.

#### PREVIOUS EFFORTS

The main problem with fluid modeling is the interpretation of modeling data. To relate Freon results to those from water tests several techniques have been developed.

The first systematic analysis to model water CHF behavior with Freon was presented by Barnett [1]. He attempted to construct a model according to the laws of dimensional analysis by taking into account only that number of parameters in a given set which would be compatible with the degrees of freedom available for experimentation. A set of fourteen parametric combinations that seemed most plausible to Barnett was explored in subsequent experimental investigations [2, 3]. This procedure showed only limited success but laid the groundwork for the more direct empirical techniques that followed.

Stevens and Kirby [4] developed a graphical correlation for CHF, and observed that both

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water and Freon CHF data for tubes would fall on the same curve if water data were multiplied by a constant factor—called the scaling factor. In their later analysis Stevens and Macbeth [5] showed that this mass flux scaling factor technique also yields accurate modeling for a range of annular and rod bundle geometry and axial power profiles.

While this successful procedure of Stevens and Kirby [4] has created widespread interest and confidence in the Freon/water modeling of CHF, it does not provide any physical basis for a calculated scaling factor. It has also the disadvantage of lacking generality as it is only valid at a single pressure with a single fluid type.

Behar [6] suggested an analytic procedure to express the Stevens and Kirby scaling factor in terms of Barnett's best dimensionless sets. Nevertheless, application of this analysis to fluids other than Freon and water was not attempted.

Staub [7] proposed a modification to the Stevens and Kirby graphical correlation and demonstrated that the scaling factor does not appreciably vary with pressure in the range tested (water equivalent 500–1500 psia).

Coffield [8] extended the application of CHF modeling to subcooled liquid conditions. He showed that the scaling factor technique was successful for subcooled CHF modeling. He also reported that Behar's [6] analysis predicted a scaling factor for water/Freon-113, approximately 30 per cent lower than that obtained from experimental data.

Groeneveld [9] developed a new optimization technique for the determination of accurate scaling factors from experimental data. His analysis showed that the mass flux scaling factor should be 1.42 (for water/Freon-12 at water equivalent pressure of 1000 psia) instead of 1.54as proposed initially by Stevens and Kirby [4]. In their recent study Stevens and Macbeth [5] have revised their earlier work and now also recommend a factor of 1.4 (for water equivalent pressure of 750–1200 psia).

Dix [10] suggested that scaling factors exhibit

effects of mass flux, length-to-diameter ratio, and pressure. However his study was based on comparing Freon-114 CHF data with a water CHF correlation proposed by Biasi [11]. As the range of parameters tested was not large, no definite conclusions can be drawn.

Kampfenkel [12] determined from experimental CHF data a functional relationship between various dimensionless groups by using the local conditions hypothesis; he used this correlation to derive scaling factors. Since the empirical function was derived from data on circular tubes, it is not applicable to annular or rod bundle geometries. Furthermore its validity for fluids other than Freon-12 and water was not examined.

Bouré [13] obtained dimensionless groups in a classical way from an arbitrarily selected set of parameters only; these groups were then amended by correction factors. Although the method is general in form, its applicability to fluids other than Freon-12 and water, and a liquid/vapor density ratio other than 70 was not investigated. No explanation was given as to how the correction factors were determined.

Tong et al. [14] showed that existing CHF correlations can be used to develop the scaling factor and suggested that the scaling factor appears to be a function of mass flux and critical pressure ratio.

Recently a comprehensive critical study of scaling laws was presented by Andersen and Pejtersen [15]. They concluded that the scaling factor method and use of Freon have so far met with only moderate success. As a result they proposed that azeotropic mixtures be used for the modeling fluid.

#### PRESENT EFFORT

Notwithstanding the conclusions of Andersen and Pejtersen [15], a generalized method for fluid modeling of CHF has been successfully developed in the present investigation by applying the principles of classical dimensional analysis and Buckingham's Pi theorem [16]. The resulting "compensated distortion model" has been tested at different liquid/vapor density ratios for various Freons, water, potassium and carbon dioxide over a large range of mass flux and inlet subcooling, and in both subcooled and quality regions. Additionally a CHF correlation scheme based on dimensionless groups has been proposed in order to test the accuracy of the modeling technique. Its efficacy has been examined in depth for annular and rod bundle geometries.

#### ANALYSIS

A model\* study should furnish a useful qualitative indication of the characteristics of the prototype,† and quantitative information of at least one main phenomenon. The theory of similitude upon which the model laws are based, may be developed by using dimensional analysis, thus establishing the principles of model design, operation, and interpretation.

At the very start, comment Rohsenow and Choi [18], it is necessary to know, or more typically to guess, the independent variables that determine the behavior of a particular dependent variable of interest. The authors add that these can usually be found by logic or intuition developed from previous experiences with problems of a similar nature, but there is no way to insure that all essential quantities have been included.

A search of the literature shows that for general flow boiling heat transfer the following variables may be selected:

The system describing parameters  $-\phi$ , G,  $\Delta H$ , L, D, g.

The primary properties  $-\lambda$ ,  $\rho_L$ ,  $\rho_V$ ,  $\mu_L$ ,  $\mu_V$ ,  $Cp_I$ ,  $Cp_V$ ,  $K_L$ ,  $K_V$ ,  $\sigma$ 

The secondary properties  $-\gamma$ ,  $\beta$ 

For a simple tube L and D are length and dia-

meter respectively. For the more complex geometries—such as annulus or rod bundle—the parameters L and D may be taken as characteristic lengths; alternatively, they may be redefined or extended.

The preceding list of variables shows a conspicuous absence of system pressure, critical pressure, saturation temperature and wall temperature. A detailed analysis would show however, that the pressure and saturation temperature are taken into account indirectly via the physical properties normally calculated at saturation temperature. Baker's [19] study on twophase flow patterns shows that the liquid/vapor density ratio is more significant than the operating/critical pressure ratio. The studies of Jens and Lottes [20] and Bernath and Begell [21] have demonstrated that in boiling flow the wall temperature is a function of the variables already included in the above list.

In view of these arguments, and if the conversion of heat energy into mechanical energy, or vice versa, is neglected, then it is reasonable to assume that for flow boiling heat transfer a relationship of the following type exists,

$$f(\phi, G, \Delta H, L, D, g, \lambda, \rho_L, \rho_V, \mu_L, \mu_V, Cp_L,$$
$$Cp_V, K_L, K_V, \sigma, \gamma, \beta) = 0.$$
(1)

According to Kay [22], with problems involving heat transfer between wall and fluid, it is appropriate to work with five fundamental dimensions: M, L, T,  $\theta$  and H. A theoretical study of dimensional analysis by Langhaar [23], has shown that the number of dimensionless products in a complete set is equal to the total number of variables minus the rank of their dimensional matrix.\* Since  $\Delta H/\lambda$ , L/D,  $\rho_V/\rho_L$ ,  $\mu_V/\mu_L$ ,  $Cp_V/Cp_L$ ,  $K_V/K_L$  are evidently dimensionless products, the six variables ( $\Delta H$ , L,  $\rho_V$ ,  $\mu_V$ ,  $Cp_V$  and  $K_V$ ) may be tentatively disregarded. Then the dimensional matrix of the variables is,

<sup>\*</sup> A device which is so related to a physical system that observations on the model may be used to predict accurately the performance of the physical system in the desired respect.

<sup>&</sup>lt;sup>†</sup> The physical system for which predictions are to be made. These definitions and others used in the present investigation are due to Murphy [17].

<sup>\*</sup> Langhaar [23] has pointed out that Buckingham's [16] rule regarding the number of dimensionless products in a complete set, is a convenient rule of thumb but is not infallible.

	φ	G	D	g	λ	$\rho_L$	$\mu_L$	Cp <sub>L</sub>	K <sub>L</sub>	σ	γ	β
М	0	1	0	0	-1	1	1	-1	0	1	-1	-1
L	-2	-2	1	1	0	-3	-1	0	-1	0	1	1
Т	-1	-1	0	-2	0	0	-1	0	-1	-2	2	2
$\theta$	0	0	0	0	0	0	0	-1	-1	0	0	1
Η	1	0	0	0	1	0	0	1	1	0	0	0

It can be shown that the fifth order determinant of this matrix is not zero, hence the rank of this matrix is five. Accordingly the matrix furnishes seven independent dimensionless products. Thus, there are thirteen dimensionless products in the complete set, and by writing a matrix of solutions they can be readily evaluated.

Consequently, by virtue of Buckingham's Pi theorem [16] and subsequent transformations, equation (1) reduces to one possible set of Pi terms, viz:

$$\pi_1 = F(\pi_2, \pi_3, \pi_4, \dots, \pi_{13})$$
 (2)

where

$$\pi_{1} = \frac{\phi}{G\lambda} \qquad \text{(Boiling number)}$$

$$\pi_{2} = \frac{GD}{\mu_{L}} \qquad \text{(Reynolds number)}$$

$$\pi_{3} = \frac{\mu_{L}^{2}}{\sigma D \rho_{L}} \qquad \text{(Weber-Reynolds number)}$$

$$\pi_{4} = \frac{\mu_{L}}{\mu_{V}} \qquad \text{(Liquid/vapor viscosity ratio)}$$

$$\pi_{5} = \frac{\Delta H}{\lambda} \qquad \text{(Subcooling number)}$$

$$\pi_{6} = \frac{\rho_{L}}{\rho_{V}} \qquad \text{(Liquid/vapor density ratio)}$$

$$\pi_{7} = \frac{L}{D} \qquad \text{(Specifies geometric similarity)}$$

$$\pi_{8} = \frac{gD^{3}\rho_{L}}{\mu_{L}^{2}} \qquad \text{(Froude-Reynolds number)}$$

$$\pi_{9} = \frac{Cp_{L}\mu_{L}}{K_{L}} \qquad \text{(Liquid Prandtl number)}$$

 $\pi_{10} = \frac{Cp_{\nu}\mu_{\nu}}{K_{\nu}} \quad \text{(Vapor Prandtl number)}$   $\pi_{11} = \frac{K_L}{K_{\nu}} \quad \text{(Liquid/vapor thermal conductivity ratio)}$   $\pi_{12} = \frac{\gamma^{\pm}\mu_L}{D\rho_L^{\pm}} \quad \text{(Barnett number)*}$   $\pi_{13} = \frac{\beta Cp_L}{\nu \lambda} \quad \text{(Saturation temperature)}$ 

number).

There are infinitely many different complete sets of dimensionless products than can be formed from equation (1). However Buckingham [16] has demonstrated, with the aid of a well chosen example, that some sets of products are more useful in practice than others. The above listed groups were chosen on the basis of the following considerations:

Each of the original independent variables that can be easily regulated experimentally  $(G, \Delta H \text{ and } L)$ , occurs in only one dimensionless product. This allows the maximum amount of experimental control over the dimensionless variables.

Most of the dimensionless groups are expressed as classical numbers (e.g. Reynolds, Prandtl, Froude, Weber, etc.) These are of a fundamental importance which extends beyond the single, isolated problem, and their physical significance has been discussed in the literature.

The dependent variable ' $\phi$ ' occurs only in one dimensionless product. This enables us to

<sup>\*</sup> This dimensionless group is named by the present author as the Barnett number in recognition of Barnett's [1] original work on scaling laws in boiling heat transfer.

know how this variable depends on other variables.

Since dimensionless groups are composed of the products and quotients of several variables, the measurement of which is subject to error, the boiling number  $(\phi/G\lambda)$  is chosen as the dependent dimensionless variable. The two components (G and  $\lambda$ ) can be determined with sufficient accuracy, hence their contribution to the error of the group will be negligible. Therefore the boiling number can be evaluated as accurately as the heat flux itself.

Equation (2) thus provides a general equation for flow boiling heat transfer. Since this equation is entirely general it applies to any system which is a function of the same variables. By extending the general equation in accordance with the theory of models outlined by Murphy [17] it follows that a true model,

 $(\pi_1)_{\text{prototype}} = (\pi_1)_{\text{model}}, \qquad (3)$ 

may be designed and operated if

$$(\pi_{2})_{\text{prototype}} = (\pi_{2})_{\text{model}}$$

$$(\pi_{3})_{\text{prototype}} = (\pi_{3})_{\text{model}}$$

$$(\pi_{13})_{\text{prototype}} = (\pi_{13})_{\text{model}}.$$

$$(4)$$

Obviously it is impracticable to satisfy all 12 design conditions. Thus a true model—in which all significant characteristics of the prototype are faithfully reproduced to scale—is almost impossible. Nevertheless it may be feasible to design an adequate model from which accurate predictions of one characteristic of the prototype namely CHF (critical heat flux) may be made, but which will not necessarily yield accurate predictions of other characteristics, e.g. pressure drop, stability, etc. Effectively, it means that those Pi terms which do not play a significant role in CHF are discarded. A short discussion on the said matter follows.

Critical heat flux experiments using liquid nitrogen to determine the influence of a positive or negative earth gravity to the flow direction have been carried out by Papell *et al.* [51]. The authors reported that beyond a certain velocity there was no longer an influence of the buoyancy orientation relative to the flow direction, and when the pressure or subcooling is increased, the liquid velocities necessary to make gravity unimportant become smaller. This is supported by earlier experiments of Lee and Obertelli [24], where it was shown that there was no appreciable difference in water CHF data between upward and downward flow. This indicates that the influence of gravity on CHF is weak for most practical cases. Thus  $\pi_8$  the Froude number can be dropped from equation (2) for the range of parameters of current interest in reactor design.

Jones and Hoffman [25] conducted CHF experiments with potassium and water and have concluded that water appears to adequately model potassium, and that it should be possible to neglect groups containing liquid thermal conductivity despite the fifty-fold difference in this property between water and potassium. A similar experimental study with boiling potassium and water (conducted in Russia) by Aladyev et al. [26] led them to conclude that the Prandtl number does not have any effect on CHF. Since specific heat, like thermal conductivity, is also a function of temperature, it is assumed that the group  $Cp_1/Cp_v$  is relatively unimportant. The validity of this assumption is substantiated by the CHF data of carbon dioxide and potassium presented in the later sections of this report. Thus it would be justifiable to eliminate liquid and vapor Prandtl numbers and the ratio of their thermal conductivity (i.e.  $\pi_9$ ,  $\pi_{10}$  and  $\pi_{11}$ ) from equation (2) on the basis of their weak influence on CHF.

The groups  $\pi_{12}$  and  $\pi_{13}$  basically consist of variables from the secondary group of properties; it is assumed that they are of secondary importance and should be included only if the primary group of properties is not adequate. Experiments on CHF with carbon dioxide at the University of British Columbia by Rotem and Hauptmann [27] support the foregoing assumption.

In view of the preceding discussions a general

CHF prediction equation can then be written as

$$\pi_1 = F(\pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7).$$
(5)

It follows that a model for predicting CHF may be designed and operated if

$$\begin{array}{c} (\pi_2)_{\text{prototype}} = (\pi_2)_{\text{model}} \\ \vdots & \vdots \\ \vdots & \vdots \\ (\pi_7)_{\text{prototype}} = (\pi_7)_{\text{model}}. \end{array}$$

$$(6)$$

Examination of equation (6) shows that it is still not practicable to satisfy all the six design conditions specified above. As a result some design conditions will be violated, whereupon

$$(\pi_1)_{\text{prototype}} \neq (\pi_1)_{\text{model}}$$

Such models by definition are distorted models. It is interesting to note Murphy's [17] comments at this juncture, that in fluid flow problems, the distorted models are the rule rather than the exception.

A standard procedure for dealing with the distorted models is to determine the 'prediction factor'  $\delta$  so that

 $(\pi_1)_{\text{prototype}} = \delta(\pi_1)_{\text{model}}.$  (7) However evaluation of such a prediction factor is difficult as it is a function of the degree of distortions and may in addition be a function of

one or more of the Pi terms. An alternative solution to this problem suggested by Murphy [17] is to compensate for an inequality in one or more dimensionless groups by introducing a controlled distortion in another, so that the prediction factor becomes unity.

In order to apply the compensated distortion technique to the problem under investigation, we first isolate those Pi terms that can be conveniently controlled in experimentation. The terms  $\pi_5$ ,  $\pi_6$  and  $\pi_7$  can be independently satisfied in both model and prototype by controlling inlet temperature, pressure and geometry.

Since pressure must be fixed in order to control the density ratio  $(\pi_6)$ , the viscosity ratio

 $(\pi_4)$  will be distorted. Consequently other Pi terms with viscosity as a parameter might also get distorted. To solve this problem of multiple distortion the remaining Pi terms are expressed as a CHF modeling parameter ( $\psi_{CHF}$ ), i.e.

$$\psi_{\rm CHF} = F(\pi_2, \pi_3, \pi_4). \tag{8}$$

Tests have shown that a functional relationship between dimensionless groups can, as a rule, be represented quite satisfactorily by a product of power functions over the range of values of practical interest, hence it is assumed that

$$\psi_{\rm CHF} = \pi_2 \times \pi_3^{n_1} \times \pi_4^{n_2}, \qquad (8a)$$

The constants  $n_1$  and  $n_2$  are the measures of distortion for  $\pi_3$  and  $\pi_4$  respectively; the resulting distortion factors between model and prototype are compensated for by adjusting the mass flux in the freely floating parameter  $\pi_2$  (the Reynolds number) in such a manner that the prediction factor becomes unity. If values of constants  $n_1$  and  $n_2$  are known then the CHF prediction equation can be rewritten as

$$\pi_1 = F(\psi_{\text{CHF}}, \Delta H/\lambda, \rho_L/\rho_V, L/D).$$
(9)

It must be distinctly realized that for the purpose of modeling, the prediction equation (9) does not require the functional relationship between the modeling parameter  $\psi_{CHF}$  and the dependent variable  $\pi_1$ . Indeed, this is the major contribution of the compensated distortion technique. It thus reduces the empiricism to a minimum degree in order to ensure accurate modeling. The constants  $n_1$  and  $n_2$  simply relate the groups  $\pi_2$ ,  $\pi_3$  and  $\pi_4$ in a common frame of reference. Since  $\pi_2$  is more amenable to experimental control, by varying mass flux, it is chosen as the reference dimensionless group.

The modeling design criteria readily follows from equation (9),

These four design conditions are now compatible with the four degrees of freedom normally available in experimentation (i.e. mass flux, inlet temperature, pressure and geometry).

With regard to the preceding analysis a statement by Langhaar [23] is worth mentioning. In discussing modeling analysis he unequivocally stated that the most important part of the work is to justify the departures from the complete similarity or to apply theoretical corrections to compensate for them. Accordingly, the present investigation has dealt with both aspects in considerable detail.

#### Determination of constants $n_1$ and $n_2$

The values of  $n_1$  and  $n_2$  in equation (8) are determined from experimental data. For determining  $n_1$  two sets of experiments are required

(1) where  $\pi_2$  is varied and  $\pi_3$  (the parameter under investigation) together with  $\pi_4$ ,  $\pi_5$  and  $\pi_6$ , are held constant, i.e.

$$(\pi_1)_{\bar{3}} = f(\pi_2, \bar{\pi}_3, \bar{\pi}_4, \bar{\pi}_5, \bar{\pi}_6)$$
 (11a)

in which the bar denotes constant values, and

(2) 
$$(\pi_1)_{\bar{3}} = f(\pi_2, \bar{\pi}_3, \bar{\pi}_4, \bar{\pi}_5, \bar{\pi}_6)$$
 (11b)

i.e.  $\pi_2$  is varied and  $\pi_3$  is held constant at another value  $\overline{\pi}_3$ , but  $\pi_4, \pi_5$  and  $\pi_6$  are held constant at previous values  $\overline{\pi}_4, \overline{\pi}_5$  and  $\overline{\pi}_6$ .

These requirements are satisfied by using existing CHF experimental data for a single fluid (e.g. Freon or water) over a series of mass flux at a fixed pressure and inlet temperature in two tubes with somewhat different diameters and lengths but constant L/D. By plotting  $\pi_1$  against  $\pi_2$  and  $\overline{\pi}_3$  and  $\overline{\pi}_3$  as parameters, the value of  $n_1$ is determined so that curves for  $\overline{\pi}_3$  and  $\overline{\pi}_3$  are collapsed into a single curve. An optimization program has yielded the value of  $n_1 = \frac{2}{3}$ .

It is important that the two sets of data are taken in geometries of neighbouring diameters; because the underlying principle in solving this problem is that of perturbation. The system is perturbed by a small amount in order to determine the effect of distortion; it is not the purpose of the above analysis to compute the effect of diameter on CHF. A comparable perturbation could be introduced in the system by changing, for example the surface tension in  $\overline{\pi}_3$  and  $\overline{\overline{\pi}}_3$  while maintaining the diameters constant in the two sets of data.

Similarly for determining  $n_2$  a further two sets of experiments are required,

where (π<sub>2</sub> × π<sup>4</sup><sub>3</sub>) is varied, and π<sub>4</sub> the parameter under investigation, together with π<sub>5</sub> and π<sub>6</sub>, are held constant, i.e.

$$(\pi_1)_{\overline{4}} = f[(\pi_2 \times \pi_3^{\frac{4}{3}}), \bar{\pi}_4, \bar{\pi}_5, \bar{\pi}_6]$$
 (12a)

in which the bar denotes constant values.

(2) 
$$(\pi_1)_{\overline{4}} = f[(\pi_2 \times \pi_3^{\frac{3}{2}}), \overline{\pi}_4, \overline{\pi}_5, \overline{\pi}_6]$$
 (12b)

i.e.  $(\pi_2 \times \pi_3^{\frac{3}{4}})$  is varied and  $\pi_4$  is held constant at another value  $\overline{\pi}_4$ , but  $\pi_5$  and  $\pi_6$  are held constant at previous values viz.  $\overline{\pi}_5$  and  $\overline{\pi}_6$ .

These requirements are satisfied by using existing CHF experimental data with two different fluids (e.g. Freon and water) at equal density ratios ( $\rho_1/\rho_V$ ), equal inlet subcooling number ( $\Delta H/\lambda$ ), and identical geometries, for a series of mass fluxes. By plotting  $\pi_1$  against ( $\pi_2 \times \pi_3^{\frac{1}{3}}$ ) and  $\bar{\pi}_4$  and  $\bar{\pi}_4$  as parameters, the value of  $n_2$  is determined so that curves for  $\bar{\pi}_4$  and  $\bar{\pi}_4$  are collapsed into a single curve. An optimization program has yielded the value of  $n_2 = -\frac{1}{5}$ .

It is to be noted that  $n_1$  and  $n_2$  were calculated from Freon-12 and water data at density ratio  $(\rho_I/\rho_V) \simeq 20$  and in simple tubular geometries only. Nevertheless the proposed modeling parameter

$$\psi_{\text{CHF}} = \left[ \left( \frac{GD}{\mu_L} \right) \times \left( \frac{\mu_L^2}{\sigma \mathbf{D} \rho_L} \right)^{\frac{3}{3}} \times \left( \frac{\mu_V}{\mu_L} \right)^{\frac{1}{3}} \right] \quad (13a)$$

is general in nature, and is applicable to different fluid types, density ratios, and geometries.

By rearranging, it can be shown that the modeling parameter is a function of classical dimensionless groups, viz



It is interesting to note that Pogson *et al.* [28] have derived a semi-analytical expression for liquid film thickness t in annular flow, as

$$\frac{t}{D} = C_1 \left(\frac{\rho_L}{\rho_V}\right)^a \times \left[ \left(\frac{G^2 D}{\rho_L \sigma}\right)^b \times \left(\frac{G D}{\mu_L}\right)^c \times \left(\frac{G D}{\mu_V}\right)^d \right] \qquad (13c)$$

which has accurately correlated the measured liquid film thickness for constant inlet sub-cooling.

Comparison of equations (13c) and (13b) shows that identical dimensionless groups have been used in both studies. Although the comparison ends here, it does lend support to the present analysis in the identification of pertinent dimensionless groups.

# RANGE OF VALIDITY AND LIMITATIONS OF MODELING PARAMETER

In spite of the generality of the proposed modeling parameter its range of validity has inherent limitations because of the fact that two empirical constants  $n_1$  and  $n_2$  have been introduced. Consequently the range of validity can be established only by testing the modeling parameter with available experimental observations.

Examination of the modeling parameter (equation (13a)) apparently shows that the model and prototype may have different cross-sectional geometries (i.e. characteristic parameter D). This of course is not so, as it should be recalled that a perturbation technique on the circular geometries was employed in determining  $n_1$ , and identical geometries were used for calculating  $n_2$ . As a result it may be possible to model circular geometries of neighboring diameters, but in cases of complex geometries the cross

section of the model and prototype should be identical for accurate modeling. Langhaar [23] has also pointed out that scale effects occur, to some extent, in nearly all model tests; the best guard against them is to build models as large as the prototype, if feasible.

To test the efficacy of the proposed modeling parameter over a wide range of variables, accurate values of fluid properties (viz. density, viscosity and surface tension) must be available. Unfortunately, accurately measured values of surface tension for many fluids are few and far between. Many formulae, both empirical and theoretical, have been proposed to determine the surface tension. Of these the better known are [29],

Mcleod's equation  $\sigma = C_2(\rho_L - \rho_V)^4$  (13d)

(where  $C_2$  is a constant for a given fluid)

and

Eötvös' equation  $\sigma = C_3 (\rho_L / M_w)^{\frac{3}{2}} (\theta + C_4)$ 

(where  $M_w$  is the molecular weight,  $\theta$  is the absolute temperature,  $C_3$  and  $C_4$  are constants). For Freon compounds the values of surface tension are generally calculated by Mcleod's equation, using the parachor of Sugden to determine the constant [30]. Although this technique satisfactorily correlates the few measured surface tension data of Freon-12 and Freon-22, the measured values themselves have considerable scatter (e.g. for Freon-12 at 32°F there is a 15 per cent difference in measured values of surface tension reported by two different sources [30]). For other Freon compounds such as Freon-21, -113 and -114 measured values of surface tension are not even available.

In the light of the preceding discussion it would not be unreasonable to assume that surface tension can be expressed as a function of liquid/vapor density ratio. Thus in order to make consistent sets of comparison between different modeling and prototype fluids, it is proposed that the Weber group  $\pi_3$  in the modeling parameter (equation (13a)) be replaced by the Barnett number  $\pi_{12}$ , as both groups contain comparable variables. An analysis similar to those described above yields an alternative form of modeling parameter.

$$\psi_{\text{CHF}} = \left[ \left( \frac{GD}{\mu_L} \right) \times \left( \frac{\gamma^{\frac{1}{2}} \mu_L}{D \rho_L^{\frac{1}{2}}} \right)^{\frac{3}{2}} \times \left( \frac{\mu_L}{\mu_V} \right)^{\frac{1}{2}} \right]. \quad (13)$$

It is to be noted that the alternative form equation (13) is recommended for reasons of practical application and for this reason only. In fundamental investigations the former equation (13a) should be used for the modeling parameter. Nevertheless computation of the properties of various fluids (Freons-21, -22, -113 and -114;  $CO_2$ , potassium and water) indicates that equations (13a) and (13) are comparable for a wide range of pressures.

#### COMPARISON BETWEEN COMPENSATED DISTORTION MODELING AND THE STEVENS AND KIRBY SCALING FACTOR TECHNIQUE

Three of the four modeling design criteria (equation (10)) are identical with those suggested by Stevens and Kirby [4]. The exception is the first criterion. In the present study it is proposed that the modeling parameter  $\psi_{CHF}$  (equation (13)) be equal in model and prototype; in the Stevens and Kirby [4] analysis it was suggested that

$$(GD^{\ddagger})_{\text{prototype}} = (K_S GD^{\ddagger})_{\text{model}}$$
 (14)

where  $K_s$  was determined from experimental data at each density ratio.

The modeling parameter (equation (13)) can be rewritten as:

$$\psi_{\rm CHF} = CGD^{\frac{1}{3}} \tag{15}$$

where

$$C = \left(\frac{\gamma}{\rho_L \mu_L}\right)^{\frac{1}{2}} \left(\frac{\mu_L}{\mu_V}\right)^{\frac{1}{2}}.$$
 (15a)

Evidently the format of the proposed modeling parameter is similar to that of Stevens and Kirby [4], in spite of the fact that they were derived by different approaches.

Furthermore, Stevens and Macbeth [5] have concluded that for circular geometries there exists much freedom in the choice of the "size scaling factor"  $F_D$  (size of the prototype/size of the model), but for complex geometries, acceptable performance is obtained only when  $F_D = 1$ ; large discrepancies occur when  $F_D$  is much greater than unity. The present analysis also recommends that cross-sectional geometry in the prototype and model must be identical, which is consistent with Stevens and Macbeth's [5] findings.

The modeling parameter also enables us to calculate the mass flux scaling factor  $F_G$  (mass flux for the prototype/mass flux for the model) for a given fluid combination and liquid/vapor density ratio,

$$F_{G} = \frac{G_{\text{prototype}}}{G_{\text{model}}} = \left\{ \frac{(\gamma/\rho_{L}\mu_{L})_{\text{model}}}{(\gamma/\rho_{L}\mu_{L})_{\text{prototype}}} \right\}^{\frac{1}{3}} \times \left\{ \frac{(\mu_{L}/\mu_{V})_{\text{model}}}{(\mu_{L}/\mu_{V})_{\text{prototype}}} \right\}^{\frac{1}{3}}.$$
 (16)

Table 1 lists the predictions of equation (16) for five different fluid combinations at several density ratios. Only those values of density ratio were chosen for which empirically determined mass flux scaling factors have been reported.

It can be seen from Table 1 that for water/ Freon-12 the predictions are consistent with Stevens and Macbeth's [5] recommendation of  $F_G = 1.4$  (for the range of 750-1200 water equivalent pressure) and Groeneveld's [9] empirical value  $F_G = 1.42$ . The predictions are also consistent with experiments of Staub [7] where it was shown that the same scaling factor is applicable to water/Freon-22 as water/Freon-12 at  $\rho_I/\rho_V = 20$ .

	De	Density ratio $\rho_L/\rho_V$ (water equiv. press. psia)							
Fluid combination	7·0 (2000)	12·0 (1500)	20-0 (1000)	29-0 (750)	41·0 (570)	980-0 (25)			
Water Freon-12	1.37	1.375	1-38	1.41	1.43				
Water Freon-22			1.38	_		_			
Water Freon-113	1-31		1'34	_	and the second sec	*****			
Water Potassium		-1960/196aa	-14 <sup>606-</sup>			1.1			
$\frac{\text{Water}}{\text{CO}_2}$		Jashum	1-2						

Table 1. Mass flux scaling factor  $F_{g}$  (Predicted by the present analysis, equation (16))

Staub [7] also concluded that there is no appreciable difference in the scaling factor for the range of  $\rho_L/\rho_V$  from 11 to 50. A similar trend is exhibited by the present analysis.

By analyzing water and potassium CHF data at low pressures ( $\rho_L/\rho_V$  from 500 to 1200), Staub [7] has recommended that for water/ potassium at low pressure the most suitable value of  $F_G = 1.0$ ; the present analysis predicts a value of  $F_G = 1.1$ .

Coffield [8] used an optimization program to calculate scaling factors for water/Freon-113 as  $F_G = 1.4$  at  $\rho_L/\rho_V = 7.0$  and  $F_G = 1.46$  at  $\rho_L/\rho_V = 20.0$ . The prediction of the present analysis ( $F_G = 1.31$  and 1.34 respectively) is in good agreement in view of the fact that in the subcooled region a 5 per cent change in  $F_G$ changes the measured to predicted ratio of CHF by only 1 per cent. The predicted trend of increase in  $F_G$  with increasing  $\rho_L/\rho_V$  is also consistent with the observations of Coffield [8].

Recently Hauptmann *et al.* [35] presented CHF data for CO<sub>2</sub> and they reported that for water/CO<sub>2</sub> at  $\rho_L/\rho_V = 20$ , the best fit value of  $F_G = 1.13$ . The present analysis predicts  $F_G =$ 

1.2 which compares well with the results of Hauptmann et al. [35].

In summary, the proposed modeling parameter  $\psi_{CHF}$  (equation (13)) is in excellent agreement with the empirically derived mass flux scaling factors of a large number of investigators for different fluids. This convincingly indicates that

- -important dimensionless groups pertinent to CHF have been included, and

#### COMPARISON WITH BOURÉ'S METHOD TO DEVELOP SIMILARITY LAWS\*

A methodical approach for determining the scaling factor was presented by Bouré [13]. The classical dimensional analysis was employed, and a set of five primary dimensionless groups was arbitrarily selected. Subsequently, three

Note: Properties at saturation line were taken from these references: water [31], Freon compounds [32], potassium [33] and  $CO_2$  [34].

<sup>\*</sup> It had not been the intention of the author to present a critical discussion on Dr. Bouré's paper [13] in relation to the proposed technique. This was undertaken at the suggestion —rather insistent—of one of the reviewers.

of these dimensionless groups were multiplied by correction factors; for the remaining two groups it was assumed that the correction factor was unity. This method implies that the correction factors take into account those dimensionless groups which had not been included in the primary list and thus depend upon the fluid, the pressure and, the geometry parameter which appear in the primary list. Recently, Bouré [52] has presented a set of scaling laws in which correction factors were applied to all five primary groups. However Bouré [52] has remarked that practically this adjustment can only be done by using correlations for both prototype and modeling fluids. Consequently this method is only as accurate as the two correlations employed, and is not applicable beyond the range of the validity of the correlations. It may be further argued that if the correlations are assumed to be accurate, then it hardly appears necessary to do modeling tests. Moreover, it is well known that CHF correlations for tubes do not accurately predict CHF for annular and rod bundle geometries. Thus the scaling factors derived from tubular data by the said technique cannot be justifiably used for complex geometries.

The primary advantage of Bouré's multiparameter correction is that it provides the possibility of improved accuracy by adjusting many parameters. Nevertheless, this sophistication may not be necessary in the practical application. It is shown by statistical analysis in the forthcoming section of this report that a single-parameter correction (in flow rate) is adequate and shows excellent agreement for various fluids, pressures and geometries. No such statistical analysis was given by Bouré [13, 52], and since the data used in evaluating the correction factors were taken by Bouré [13] from an unpublished source, an evaluation of the comparative accuracy of Bouré's method could not be made.

The most serious limitation of Bouré's technique is that the correction factors are only valid for a single combination of prototype and modeling fluid, and for the pressure levels and geometries which were used in evaluating these factors. In comparison, the present technique is applicable to different fluid combinations, pressures and, simple as well as complex geometries.

Furthermore, the arbitrary selection of dimensionless groups and its subsequent amendment by correction factors (without any consideration to the experimentally observed trends of different parameters) renders the dimensional analysis redundant. Essentially, the Bouré method [13] becomes a plot of power versus mass flux (with inlet subcooling as parameter) for the prototype and modeling fluids; homologous curves are then obtained by a series of transformations. In the geometrical interpretation, the translation of the x-axis corresponds to the mass flux correction factor and of the y-axis to the power correction factor, whereas the rotation of the axes corresponds to the inlet subcooling correction factor. As a result, Bouré's technique yields a purely empirical fit of prototype and modeling data. For this purpose the technique of dimensional analysis is not even required, as shown by Groeneveld [9]. Thus the Froude number and other dimensionless groups used in Bouré's similarity laws are not physically significant.

In comparison, the present technique employs the dimensional analysis in its true sense. Pertinent dimensionless groups are identified on the basis of experimental evidence. The correction factors are applied to and by dimensionless groups as a means of compensating the distortions. Consequently the proposed model has a physical basis and offers greater generality. For example, the qualitative effect of surface tension on critical heat flux could be deduced from the present analysis (Fig. 1) which is in agreement with the experimentally observed trends. A similar attempt to determine the effect of g (acceleration due to gravity) on CHF from Bouré's analysis (Fig. 1 of Ref. [13]) shows a trend which does not agree with the experiments of Papell et al. [51]. This further



FIG. 1. Comparison of water, CO<sub>2</sub> and Freon-12 data (in tubes) at water equivalent pressure 1000 lb/in<sup>2</sup> abs. ( $\rho_L/\rho_V = 20$ )

indicates that the dimensionless groups used in Bouré's technique are not relevant to the physics of the problem. However Groeneveld's and Bouré's empirical techniques are extremely useful for those modeling fluids whose physical properties are not well known.

#### COMPARISON OF THE MODELING TECHNIQUE WITH EXPERIMENTAL DATA

Examination of the proposed modeling technique (equations (9) and (10)) shows that if  $\rho_L/\rho_V$ ,  $\Delta H/\lambda$  and L/D are the same for model and prototype, both sets of CHF data drawn in the plane of  $\phi/G\lambda$  vs.  $\psi_{CHF}$  (the modeling parameter) should plot on the same curve.

Figure 1 shows such a plot for CO<sub>2</sub>, Freon-12 and water CHF data taken in uniformly heated tubes at a liquid/vapor density ratio  $\rho_L/\rho_V = 20$ . Further plots are shown in Fig. 2 (CO<sub>2</sub> and Freon-12 at  $\rho_L/\rho_V = 12$ ), Fig. 3 (water and Freon-12 at  $\rho_L/\rho_V = 41$ ), Fig. 4 (water and Freon-113 subcooled CHF data at  $\rho_L/\rho_V = 7$ ), Fig. 5 (water and Freon-113 subcooled CHF



FIG. 2. Comparison of Freon-12 and CO<sub>2</sub> data (in tubes) at water equivalent pressure 1500 lb/in<sup>2</sup> abs. ( $\rho_L/\rho_V = 12$ )



FIG. 3. Comparison of water and Freon-12 data (in tubes) at water equivalent pressure 570 lb/in<sup>2</sup> abs. ( $\rho_L/\rho_V = 41$ ).



FIG. 4. Comparison of water and Freon-113 subcooled data (in tubes) at water equivalent pressure 2000 lb/in<sup>2</sup> abs. ( $\rho_L/\rho_V = 7$ ).

data at  $\rho_L/\rho_V = 20$ ), and Fig. 6 (water and potassium at  $\rho_L/\rho_V = 980$ ). It can be seen that in all cases the modeling and prototype CHF data does indeed fall on the same curve (obtained from best eye-fit). This indicates the success of the modeling technique with different types of fluids at various density ratios, and thus establishes the generality of the present technique. The proposed technique has also been successfully tested for Freons-11, -21 and -114. However they are not shown here in order to avoid graphical plethora.

Some scatter in Figs. 1–6 are the results of the fact that CHF data in matching tubular geometries were not available, and in some cases corrections for  $\Delta H/\lambda$  and L/D also had to be made.

For those cases where CHF data in identical geometries (for model and prototype) are avail-



FIG. 5. Comparison of water and Freon-113 subcooled data (in tubes) at water equivalent pressure 1000 lb/in<sup>2</sup> abs. ( $\rho_L/$  $\rho_V = 20$ ).



FIG. 6. Comparison of water and potassium data (in tubes) at water equivalent pressure 24  $lb/in^2$  abs. ( $\rho_L/\rho_V = 980$ ).



FIG. 7. Comparison of water and Freon-12 data (in annulus) at water equivalent pressure 1000 lb/in<sup>2</sup> abs. ( $\rho_I / \rho_V = 20$ ).



FIG. 8. Comparison of water and Freon-12 data (in annulus) at water equivalent pressure 750 lb/in<sup>2</sup> abs. ( $\rho_L/\rho_V = 29$ ).

able, the present technique models with a high degree of accuracy; this is shown in Figs. 7 and 8 where CHF data are plotted for water [42] and Freon [43] taken in uniformly heated 12 ft long annulus at  $\rho_I/\rho_V = 20$  and  $\rho_L/\rho_V = 29$ .

The present model predicts mass flux scaling factors almost identical to that recommended by Stevens and Macbeth [5] for the range of water equivalent pressure of 750–1200 psia. By inference therefore the present technique is also applicable to the entire range of complex geometries—uniformly and non-uniformly heated annulus, 19 and 37 rod clusters, etc.—as employed by Stevens and Macbeth [5].

#### APPLICATION OF A CORRELATION FUNCTION TO THE MODELING ANALYSIS

The graphical plots shown in Figs. 1–8, or those used by Stevens and Kirby [4], have successfully demonstrated the efficacy of the modeling techniques; nevertheless they have been criticized for inadvertently suppressing the observed errors. Estimates of modeling accuracy are further complicated by the presence of a large scatter in prototype data such as water, and by the many corrections introduced to obtain matching parameters for the model and the prototype.

Since the objective in doing CHF experiments with modeling fluids is to develop a design equation which should be capable of accurately predicting the CHF behavior of the prototype fluids, it is proposed to develop a correlating function from the general equation (9) by using modeling data only. Subsequent comparison of such a correlation with the prototype data would obviously demonstrate the usefulness and the efficacy of the modeling technique.

### Development of a correlation function

The effect of complex geometries can be incorporated by using the heated equivalent diameter. Thus the general CHF prediction equation (9) can be rewritten as

$$\frac{\phi}{G\lambda}F_1(\psi_{\text{CHF}},\Delta H/\lambda,\rho_L/\rho_V,L/D,D_{he}/D).$$
 (17)

Examination of experimental data shows that for uniformly distributed heat flux, CHF generally occurs at the end of the heated section. Hence it would be appropriate to express critical power as the dependent variable, which can be written as the dimensionless group,

$$W (Power number) = \frac{4\phi L}{G\lambda D_{he}}$$
(18a)

and equation (17) can be arranged as,

$$W = \frac{4\phi}{G\lambda} \frac{L}{D_{he}} = F_2(\psi_{\text{CHF}}, \Delta H/\lambda, \rho_L/\rho_V, L/D, D_{he}/D).$$
(18)

A simple heat balance, for a uniformly heated channel gives

$$W = X_0 + \Delta H / \lambda. \tag{19a}$$

It has been observed that at higher outlet qualities CHF primarily occurs due to depletion of the liquid film from the heated rod surface. Therefore the maximum value of the critical quality for the ideal case, where all liquid is available for cooling, can be written as,

$$X_{0, \max} = 1.$$
 (19b)

This physically signifies that independent of other parameters, CHF would occur at  $X_0 = 1$ . It follows from equation (19a) that the maximum power number

$$W_{\rm max} = 1 + \Delta H / \lambda. \tag{19c}$$

We can then define a reduced power number

$$W^{+} = \frac{W}{W_{\text{max}}} = \frac{4\phi L/G\lambda D_{he}}{1 + \Delta H/\lambda} \simeq \frac{X_{0} - X_{in}}{1 - X_{in}} \qquad (20)$$

and equation (17) can be rearranged as,

$$W^{+} = F_{3}(\psi_{\text{CHF}}, \Delta H/\lambda, \rho_{L}/\rho_{V}, L/D, D_{he}/D). \quad (21)$$

The main advantage of selecting  $W^+$  as the dependent variable lies in its inherent constraints, which are expressed as,

$$0 \leqslant W^+ \leqslant 1. \tag{21a}$$

This provides a convenient means of plotting

both subcooled and quality CHF data, and simplifies the development of the correlating function.

The following functional form for equation (21) is proposed, which satisfies the boundary conditions of inequality (21a), and follows the observed trend of data,

$$W^{+} = \{1 - \exp(-y^{+})\}$$
(22)

where

$$y^{+} = F(\psi_{\text{CHF}}, \Delta H/\lambda, \rho_{L}/\rho_{V}, L/D, D_{he}/D). \quad (22a)$$

In most cases for a limited range, equation (22a) can be expressed as a product of power functions, i.e.

$$y^{+} = K\psi^{a}_{CHF} (1 + \Delta H/\lambda)^{b} (\rho_{L}/\rho_{V} - 1)^{c} (L/D)^{d} (D_{he}/D)^{e}.$$
(22b)

K and a-e are constant which are to be determined from modeling CHF data. In some cases however a more complex function for individual groups may become necessary.

## Application of the proposed dimensionless correlation

Circular geometry. Freon-12 data for round tubes at  $\rho_L/\rho_V = 20$  from reference [36] were first used to determine the constants in equations (22) and (22b) by simply plotting data on a log-log graph. The resulting correlation is as follows

$$W^{+} = \{1 - \exp(-0.522/E^{+})\}$$
(23)

$$E^+ = E/(1 + \Delta H/\lambda)^{E/2}$$
(23a)

$$E = \psi_{\rm CHF} / (L/D)^{\sharp} . \tag{23b}$$

The correlation function (equation (23))\* is

\* It was remarked by a reviewer that, "The boiling length effect (heated length in the author's paper) is more simply and satisfactorily modeled by the modification given in [7] (by Staub) than it is using the author's equation (23)."

In the author's opinion this argument is fallacious. The modeling criterion as given in this analysis is governed by general equation (9)—not by equation (23), which is simply a specific arrangement of equation (9). To further elucidate this point, Staub's [7] boiling length correlation

$$X_{0} = F\left[\psi_{CHF}\left(\frac{D}{L_{b}}\right)^{0.58}\right]$$
(9a)

could easily be written in terms of basic parameters as,

$$\left(\frac{4\phi L}{G\lambda D} - \frac{\Delta H}{\lambda}\right) = F\left[\psi_{CHF}\left(\frac{D}{L}\right)^{0.58} \left\{1 + (\Delta H/\lambda) \left| \left(\frac{4\phi L}{G\lambda D} - \frac{\Delta H}{\lambda}\right) \right\}^{0.58}\right]$$
(9b)

since

$$\frac{4\phi L}{G\lambda D} - \frac{\Delta H}{\lambda} \simeq X_0 \tag{9c}$$

and

$$\left(\frac{D}{L_b}\right) = \left(\frac{D}{L}\right) \left(1 + \frac{\Delta H/\lambda}{X_0}\right).$$
 (9d)

It is readily apparent that Staub's correlation, equation (9b), is merely another arrangement of the general equation (9) and, is based on the same modeling criterion as equation (23).

Furthermore, from the correlation point of view, Staub's [7] modification is neither more simple nor satisfactory. Effectively, Staub's replacement of heated length L by the boiling length  $L_b$  amounts to multiplying (D/L) by  $(1 + (\Delta H/\lambda)/X_0)$  as shown above (9b)-(9d). Subsequently, in order to compare with Staub's correlation (i.e. equation 9b), the proposed correlation equation (23) is rewritten in a similar format,

$$\left(\frac{4\phi L}{G\lambda D}\right) / (1 + \Delta H/\lambda) = F \left[\psi_{CHF} \left(\frac{D}{L}\right)^{0.6} \right]$$

$$(1 + \Delta H/\lambda)^{0.5\psi_{CHF}(D/L)^{0.6}}$$

$$(23c)$$

Clearly, equations (9b) and (23c) have a similar degree of simplicity. Staub's correlation may appear simple, if it is written in terms of the outlet quality  $X_0$ . But it is to be remembered that  $X_0$  is an artificial parameter, and is actually calculated from equation (9c).

On the other hand, Staub's correlating scheme has certain extremely unsatisfactory features:

- (i) the correlation function is implicit, and thus involves iteration in predicting critical heat flux.
- (ii) the correlation involves plotting the function  $X_0$  against  $(1 + [(\Delta H/\lambda)/X_0])$ ; Colburn [53] has very lucidly warned against plotting a function against itself.
- (iii) the correlation give progressively larger error as  $X_0$  decreases and becomes meaningless when  $X_0 \leq 0$ ; Coffield [8] has pointed out this severe limitation of Staub's modification and has concluded that it is not applicable to subcooled CHF cases.

In contrast, the proposed scheme e.g. equation (23) does not suffer from any of the said deficiencies and, is by far more satisfactory than Staub's correlation:

- (i) it is explicit and requires no iteration.
- (ii) it is thus free from parametric distortion as the dependent variable 'φ' appears only on the left-hand side of the equation.
- (iii) it is applicable to both subcooled and quality CHF data and does not become meaningless at  $X_0 \leq 0$ .

shown as the solid line in Fig. 9; CHF data for water [44 and 45], Freon-22 [7] and Freon-12 [36] are also plotted. Figure 9 clearly demon-



FIG. 9. Comparison of water, Freon-22 and Freon-12 data (in tubes) with prediction from a dimensionless correlation based on Freon-12 data, at water equivalent pressure  $1000 \text{ lb/in}^2$  abs. ( $\rho_L/\rho_V = 20$ ).

strates that a correlation developed from Freon-12 data only is in excellent agreement with water and Freon-22 data.

It can be shown that for zero inlet subcooling,

$$V^+ \to X_0 \tag{24}$$

$$E^{+} \rightarrow E \left\{ = \left(\frac{\gamma}{\mu_{L}\rho_{L}}\right)^{\frac{1}{2}} \times \left(\frac{\mu_{L}}{\mu_{V}}\right)^{\frac{1}{2}} \times GD^{\frac{1}{2}} \times (D/L)^{0.6} \right\}.$$
(25)

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Interestingly enough, the parameters  $X_0$ , and a dimensional form of parameter E were used by Stevens and Kirby [4] in their graphical correlation.

Nevertheless, it must be emphasized that the first objective of this analysis was not to develop a CHF correlation *per se*. Moreover the proposed modeling technique is neither derived from the correlation, nor is dependent upon it. The correlation scheme has been developed in order to provide a simple and practical device for determining the accuracy of the proposed modeling technique. Application of such a scheme to annular and bundle geometry data follows.

Annular geometry. Ahmad and Groeneveld [46] obtained a CHF correlation for annular geometry similar to equation (22) from their 418 Freon data covering a wide range of pressure, inlet subcooling and mass flux. The rms error was 6 per cent. The authors subse-

quently applied this correlation to published 233 water CHF data, which fitted all data within  $\pm 15$  per cent with an rms error of only 8 per cent. This correlation was also compared with Barnett [47] correlation, developed from 724 reliable experimental CHF data for water flowing in annular geometries at 1000 psia, with an rms error of 5.9 per cent. As a basis for comparing the two correlations, a set of water CHF data [48] was selected which was neither used in the development of Barnett's water correlation nor for the Freon correlation of Ahmad and Groeneveld [46]. The comparison is shown in Table 2. It shows that

Table 2. Comparison between Barnett's [47] correlation and equation (22) (Taken from [46])1000 psia water data of ref. [48] as reported in ref. [49] Shroud dia. = 0.607, Rod dia. = 1.053:Length = 72 in.

	Mass	Inlet	Measured	% Error in predicting CHF from		
Run no.	$ \begin{pmatrix} 10^6 \text{lb} \\ \hline \text{h.ft}^2 \end{pmatrix} $	sub-cooling (Btu/lb)	$\left(\frac{10^6 \text{Btu}}{\text{h.ft}^2}\right)$	Barnett's [47] correlation	correlation equation (22)	
759	0.498	93	0.521	- 14.3	-13.4	
760	0.499	2	0.398	14.7	-11.7	
761	0.503	176	0.637	- 14.0	-16.7	
762	0.513	38	0.470	- 10.2	- 7.5	
763	0.514	93	0.520	- 16.3	-15.0	
764	0.522	114	0.564	- 13.9	-13.4	
765	0.523	4	0.401	- 16.9	-13.2	
766	0.528	147	0.643	- 8.7	- 9.5	
767	0.529	66	0.491	- 16.4	- 13.9	
768	0.533	36	0.455	15-1	-11.7	
769	0.793	72	0.653	- 5.3	0	
770	0.808	113	0.722	8.3	- 4.7	
771	0.816	44	0.613	3.4	2.8	
772	0.991	78	0.757	0.5	5.1	
773	1.007	117	0.886	0.2	3.8	
774	1.021	51	0.718	2.6	8.9	
775	1.477	14	0.718	9.2	14.8	
776	1.479	75	0.889	3.6	9.1	
777	1.480	119	1.011	0.2	3.9	
778	1.483	45	0.813	6.9	12.9	
779	1.489	128	1.050	1.1	3.9	
780	1.493	39	0.792	6.8	12.9	
781	1.930	113	1.064	1.9	1.6	
782	1.971	73	0.961	3.5	8.8	
783	2.419	57	0.942	3.3	8.7	
784	2.441	17	0.771	7.1	10.8	
785	2.480	120	1.141	7.2	- 4.5	
Total number o	f data 27			Maximum difference between two correlations $\rightarrow 6\%$		

- (a) the error trends for the two correlations are remarkably similar, and
- (b) the maximum difference is predicted CHF between the two correlations is only 6 per cent.

Accuracy of the proposed modeling technique is self-evident.

Bundle geometry. CHF tests made in two different laboratories on the same 9-ft, 18element bundle geometry are reported by McPherson and Ahmad [50]. Coolants were water and Freon-12, and a wide range of pressure, inlet subcooling and mass flux were covered. The authors fitted an equation (22) type of correlation to the Freon data with an rms error of 5.9 per cent and error deviations within  $\pm 15$  per cent. The predictions of this correlation were then compared with water data. Figure 10 shows that the rms error was



FIG. 10. Measured vs. predicted  $W^+$ . Pressure range: 500–1200 lb/in<sup>2</sup> abs., Inlet subcooling: 4–212°F. (Taken from reference [50]).

only 61 per cent with an average deviation of  $\pm 10$  per cent for 90 per cent of the water data and  $\pm 15$  per cent for all water data. The error distribution shown in the histograms are almost identical for Freon and water; the rms errors are also very close. This good agreement of the correlation with the water data is notable because no reference to water was used in the development of the empirical correlation. Accurate modeling of CHF in a rod bundle geometry has thus been clearly demonstrated.

#### CONCLUSIONS

- (1) A generalized CHF modeling technique developed from classical dimensional analysis has been successfully applied to many fluids at various liquid/vapor density ratios in simple as well as complex geometries.
- (2) It is shown that the proposed modeling technique is consistent with the empirical relationships of a large number of investigators.
- (3) It is demonstrated that CHF correlations obtained solely from modeling experiments using full size models accurately predict CHF for prototypes. The limitations imposed on the accuracy of this technique are those of experimentation and the correlation function used.

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#### MODELISATION FLUIDE-FLUIDE DU FLUX THERMIQUE CRITIQUE: UN MODELE DE DISTORSION COMPENSEE

Résumé—La modélisation est d'une grande importance pratique par la réduction du grand coût des essais de flux thermique critique pour des réacteurs de puissance à eau bouillante. Une méthode largement utilisée est de remplacer l'eau par un fluide ayant une plus faible chaleur latente de vaporisation. Dans cet article, une technique généralisée pour une telle similitude est développée à partir de l'analyse dimensionnelle et de la théorie classique des modèles. On a résolu le problème de la distorsion multiple en introduisant un paramètre de modélisation qui est fonction du nombre de Weber et des nombres de Reynolds superficiels du liquide et de la vapeur. La technique de distorsion compensée est utilisée dans le développement du paramètre de modélisation.

Le modèle résultant a été testé pour différents rapports de densité liquide/vapeur pour plusieurs composants Fréon, eau, potassium et dioxyde de carbone, dans un large domaine de flux massique et de sousrefroidissement à l'entrée et à la fois dans les régions sous-refroidies et de qualité. Un excellent accord existe entre les résultats expérimentaux et la technique de modélisation pour des géométries annulaires, circulaires et des faisceaux de barreaux.

#### ÄHNLICHKEITSBETRACHTUNG FÜR DEN KRITISCHEN WÄRMESTROM VON FLÜSSIGKEIT ZU FLÜSSIGKEIT: MODELL MIT KOMPENSIERTER DEFORMATION

Zusammenfassung-Modellmethoden haben ihre praktische Bedeutung in einer Kostenminderung bei der Untersuchung des kritischen Wärmestromes von Siedewasserreaktoren. Eine verbreitete Methode besteht darin, Wasser durch eine Modellflüssigkeit mit geringerer Verdampfungswärme zu ersetzen. In dieser Arbeit wurde eine allgemeine Technik zum Gebrauch einer Modellflüssigkeit entwickelt aufgrund der klassischen Dimensionsanalyse und Ähnlichkeitstheorie. Das Problem einer mehrfachen Deformation wurde durch Einführung eines Modell-Parameters gelöst, der eine Funktion der Weber-Zahl und der Flüssigkeits- und Dampf-Oberflächen-Reynolds-Zahlen ist. Die Technik der kompensierten Deformation wird angewandt zur Aufstellung des Modell-Parameters. Das resultierende Modell wurde geprüft bei

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verschiedenen Flüssigkeit-Dampf Dichteverhältnissen für einige Freon Gemische, Wasser, Kalium und Kohlendioxid, über einen weiten Bereich des Wärmestromes und der Eintrittsunterkühlung, sowohl im Unterkühlungs- wie im Sättigungsbereich. Es zeigte sich eine ausgezeichnete Übereinstimmung zwischen Versuchsdaten und der Modelltechnik für Kreis-, Ringspalt- und Stabbündelgeometrien.

#### МОДЕЛИРОВАНИЕ КРИТИЧЕСКОГО ТЕПЛОВОГО ПОТОКА ОТ ЖИДКОСТИ К ЖИДКОСТИ. МОДЕЛЬ КОМПЕНСИРОВАННОГО ПРЕОБРАЗОВАНИЯ

Аннотация—Моделирование имеет большое практическое значение для сокращения расходов на испытания критического теплового потока в кипящих ядерных реакторах. Широко используется метод замены воды модельной жидкостью с более низкой скрытой теплотой парообразования. В данной работе обобщенная методика такого моделирования критического теплового потока развивается, исходя из классической теории размерностей и теории моделирования. Задача многократного преобразования решается с помощью введения параметра моделирования, который зависит от числа Вебера и сверхкритических значений числа Рейнольдса для жидкости и пара. При выюде параметра моделирования использовалась методика компенсированного преобразования.

Полученная модель исследовалась при различных отношениях плотностей жидкости и пара для различных соединений фреона, воды, калия и двуокиси углерода в широком диацазоне значений удельного потока массы и недогрева на входе, а также как в зоне недогрева, так и в зоне насыщения. Экспериментальные данные и результаты моделирования хорошо согласуются в случаях круглой и кольцевой конфигураций и пучка стержней.